# Event-triggered Robust Sliding Mode Control for Linear Delayed Differential Systems

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**Abstract:** This paper is concerned with the global robust stability problem for linear delayed differential systems with uncertainties by using an event-triggered sliding mode control. First, a sliding mode control with event triggering scheme and the estimations of the practical sliding band which the trajectory remains at last are provided. Then Zeno phenomenon for proposed event-triggered scheme is excluded in this paper. At last, a numerical example is given to illustrate the effectiveness of our results.

## **1. Introduction**

Event-triggered control strategy gains more and more attention since it can improve the control efficiency and reduce the burden of communication or actuation in control process. It does not update in a periodic manner which is the way in the classical sampled data system. There have been a lot of research literature studying the event-triggered control, see [1]–[5]. Most of these literature investigate the systems without uncertainties since the uncertainties may influence the performance of the event-triggered control [6].

Sliding mode control (SMC) is an effective robust control strategy for hybrid or uncertain systems. It utilizes a discontinuous control to force the state trajectories of the system to some specific sliding surfaces. The sliding mode control has been applied to uncertain systems [7], time-delay systems [8], fuzzy systems [9], [10] and so on. Therefore sliding mode control with event-triggered strategy has been studied recently [11]–[13]. Since the event-triggered control strategy is a discrete control scheme, it is not possible for the system to be in ideal sliding mode, which means the trajectory can only remain in the vicinity of sliding manifold. [13] introduced the definition of practical sliding mode and proposed a global event-triggering sliding mode control for a linear time-invariant system.

In this paper, implementation of SMC with the event-triggering strategy for a linear delayed differential system. To the best of our knowledge, there has been no result of such research and it still remains chanllenging. The main contribution of this paper are highlighted as follows: 1) The system model we investigate is very comprehensive that it contains time delays, exogenous disturbance as well as event-triggered sliding mode control. 2) An event-triggered sliding mode control is designed effectively for the linear delayed differential system to force the trajectories to remain in a vicinity of the sliding manifold. 3) Zeno phenomenon is excluded under some sufficient conditions.

This paper is organized as follows. In Section 2, we give the formulates the problem of eventtriggered robust sliding mode control for linear delayed differential systems with exogenous disturbance and introduce some basic definitions and lemmas. In Section 3, the main results are presented where criteria about the trajectory is attracted towards the sliding manifold and stays within a band are established. Then a numerical simulation for illustrating the theoretical results is given in Section 5, following by conclusions in Section 6.

Notations:  $\mathbb{R}^n$  and  $\mathbb{R}^{m \times n}$  denote the *n*-dimensional Euclidean space and the set of  $m \times n$  real matrices, respectively.  $\mathbb{R}_+ = [0, \infty)$ .  $I_m$  represents the identity matrix of order *m*. For a real

symmetric matrix B,  $\lambda_{\max}(B)$  ( $\lambda_{\min}(B)$ ) denotes the maximum (minimum) eigenvalue of B. A function  $\gamma: \mathbb{R}_+ \to \mathbb{R}_+$  is a  $\mathcal{K}$ -function if it is continuous, strictly increasing and  $\gamma(0) = 0$ ; it is a  $\mathcal{K}_{\gamma}$ -function if it is a  $\mathcal{K}$ -function and also  $\gamma(s) \to \infty$  as  $s \to \infty$ .

#### 2. Problem Formulation

In this section, we state the problem formulation and present some necessary preliminaries. Consider the following linear delayed differential system with delay

$$\dot{x}(t) = Ax(t) + \alpha x(t-\tau) + B(u(t) + d(t)), \quad x_0 = x(t_0),$$
(1)

where  $x(t) \in \mathbb{R}^n$  and  $u(t) \in \mathbb{R}$  represent the state of the system and the control input, respectively.  $\tau$  denotes the delay of the system,  $d(t) \in \mathbb{R}$  is an unknown exogenous disturbance but bounded for all time, that is,  $\sup |d(t)| \le d_{\max}$ ,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^n$  and  $\alpha \in \mathbb{R}$ .

The sliding variable is designed as  $s(t) = c^T x(t)$  for  $c \in \mathbb{R}^n$ . Define the sliding manifold as

$$\mathcal{S} \triangleq \{ x \in \mathbb{R}^n \mid s = c^T x = 0 \}, \tag{2}$$

where  $c = [c_1^T, 1]^T$  with  $c_1 \in \mathbb{R}^{n-1}$ . Our purpose is to bring the trajectories of the delayed differential system (1) to the sliding manifold in finite time and the state is forced to stay there for all time. In view of (1) and (2), we have

$$\dot{s}(t) = c^{T} A x(t) + \alpha c^{T} x(t-\tau) + c^{T} B u(t) + c^{T} B d(t)$$

Now we give a useful definition and two lemmas.

**Definition 1.** [13] Let  $x(t, x_0)$  be the trajectory of the system starting from initial condition  $x_0 = x(t_0, x(t_0))$  and  $t > t_0$ . Consider the sliding manifold given by S. The system is said to be in practical sliding mode if, given any positive constant  $\Delta$ , there exists a finite time  $t_1 \in [t_0, \infty)$  such that the system trajectories reach the region in the vicinity of the sliding manifold S bounded by  $\Delta$  in time  $t_1$  and remain there for all time  $t \ge t_1$ . The region in the vicinity of the sliding manifold where the system trajectories are confined is called practical sliding band. The practical sliding mode is called ideal sliding mode if  $\Delta = 0$ .

**Lemma 1** (Halanay inequality). [14] Let w(t) be a nonnegative function defined on the interval  $[t_0 - \tau, \infty)$ , and be continuous on the subinterval  $[t_0, \infty)$ . If there exist two positive constants  $\xi$ ,  $\eta$  satisfying  $\xi > \eta$  such that  $\dot{w}(t) \le -\xi w(t) + \eta w(t - \tau)$ ,  $t \ge t_0$ . Then  $w(t) \le \sup_{\theta \in [t_0 - \tau, t_0]} w(\theta) e^{-\gamma(t - t_0)}$ ,  $\gamma > 0$  is

the smallest real root of the equation  $\xi - \gamma - \eta e^{\gamma \tau} = 0$ .

**Lemma 2.**  $\alpha + \beta > 0$ ,  $\gamma > 0$ ,  $\delta > 0$ , d > 0. Let  $g:[t_0,\infty) \to \mathbb{R}_+$  satisfy the following delayed differential inequality

$$\dot{g}(t) \le \alpha g(t) + \beta g(t-\tau) + \gamma, \quad t \in [t_0, \infty).$$
(3)

Then we have

$$g(t) \leq \left[\sup_{\theta \in [t_0 - \tau, t_0]} g(\theta) + \frac{\gamma}{\alpha + \beta}\right] e^{(\alpha + \beta)(t - t_0)} - \frac{\gamma}{\alpha + \beta}, \quad t \in [t_0, \infty).$$
(4)

**Proof.** Claim that  $y = [\sup_{\theta \in [t_0 - \tau, t_0]} y(\theta) + \frac{\gamma}{\alpha + \beta}] e^{(\alpha + \beta)(t - t_0)} - \frac{\gamma}{\alpha + \beta}$  is a solution of the delayed differential

equation

$$\dot{y}(t) = \alpha y(t) + \sup_{t \in [t-\tau,t]} \beta y(t-\tau) + \gamma, \quad t \ge t_0$$
(5)

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with the initial condition y(t) = g(t),  $t \in [t_0 - \tau, t_0]$ . To prove the claim, we check that

$$\dot{y}(t) = (\alpha + \beta) [\sup_{\substack{\theta \in [t_0 - \tau, t_0]}} y(\theta) + \frac{\gamma}{\alpha + \beta}] e^{(\alpha + \beta)(t - t_0)},$$
  
$$\sup_{\substack{\theta \in [t - \tau, t]}} y(\theta) = y(t).$$

Compared with (5), we conclude that  $y = [\sup_{\theta \in [t_0 - \tau, t_0]} y(\theta) + \frac{\gamma}{\alpha + \beta}]e^{(\alpha + \beta)(t - t_0)} - \frac{\gamma}{\alpha + \beta}$  is indeed a solution

of (5). According to [14], we have 
$$g(t) \le y(t) = (\alpha + \beta) [\sup_{\theta \in [t_0 - \tau, t_0]} y(\theta) + \frac{\gamma}{\alpha + \beta}] e^{(\alpha + \beta)(t - t_0)}$$

The proof is complete.

#### 3. Main Results

In this section, the main results of the paper on the event-triggered sliding mode control for the delayed differential system (1). The paper [13] introduced a triggering scheme for a linear time-invariant system to be globally robustly stable. In this paper, assume that  $\{t_i\}_{i=0}^{\infty}$   $(t_0 = 0)$  is the event-time sequence and we propose similarly as follows:

$$t_{i+1} = \inf\{t > t_i : \|c\| \|A\| \|e(t)\| \ge \sigma(\|x(t_i)\| + \beta)\},$$
(6)

for any given  $\beta \in (0,\infty)$  and  $\sigma \in (0,1)$ . Define the error  $e(t) = x(t_i) - x(t)$  with  $e(t_i) = 0$  for  $t \in [t_i, t_{i+1})$ . From (6), it is clearly to know that this event-triggering time sequence is dependent on the sampled state and we will show that under this global triggering scheme, the trajectories of the state remain bounded within a sliding band in the vicinity of the sliding manifold (2). The event-triggered sliding mode control is defined as

$$u(t) = -(c^T B)^{-1} (c^T A x(t_i) + K(x(t_i)) \operatorname{sign}(t_i)), \quad t \in [t_i, t_{i+1}),$$
(7)

where sign denotes the signum function and the gain *K* is a function of  $x(t_i)$  which is sampled at every event-triggering instants  $t_i$ ,  $i \in \mathbb{N}$ . Now we are in the position to give the following theorem to see the system in practical sliding mode.

**Theorem 1.** Consider the delayed differential system (1) based on the event-triggered strategy (6) and control law (7). Then if  $K(x(t_i))$  is designed as

$$K(x(t_i)) > \sup_{t \ge 0} |c^T B| d(t) + \mu(||x(t_i)|| + \beta), \quad \mu > (||c|| + \frac{1}{||A||})(|\alpha| + \eta) + \sigma,$$
(8)

the trajectory will remain within a band

$$\{x \in \mathbb{R}^{n} : |c^{T}x| \leq (||x(t_{i})|| + \beta) ||A||^{-1}\}, \quad \forall t \geq t_{0}, \eta > 0.$$
(9)

**Proof.** Construct the following Lyapunov function  $V(t) = s^2(t)/2$ . Calculate the derivative of V(t) along the trajectory of system (1), from the fact that  $e(t) = x(t_i) - x(t)$ , we have

$$\dot{V}(t) = s(t)c^{T}Ae(t) + \alpha s(t)s(t-\tau) - s(t)K(x(t_{i}))\operatorname{sign}(s(t_{i})) + s(t)c^{T}Bd(t)$$
  
$$\leq |s(t)||c^{T}Ae(t)| + \frac{|\alpha|}{2}s^{2}(t) + \frac{|\alpha|}{2}s^{2}(t-\tau) - s(t)K(x(t_{i}))\operatorname{sign}(s(t_{i}))n + |s(t)||c^{T}B|d_{\max}$$

In view of the event-triggering scheme, we note that

$$|c^{T}Ae(t)| \le ||c|| ||A|| ||e(t)|| \le \sigma(||x(t_{i})|| + \beta).$$
(10)

If the trajectories of the system start from the region where  $sign(s(t_i)) = sign(s(t))$ , then from (8) it can be derived that

$$\dot{V}(t) \leq \sigma(||x(t_i)|| + \beta) |s(t)| + \frac{|\alpha|}{2} s^2(t) + \frac{|\alpha|}{2} s^2(t - \tau) - |s(t)| K(x(t_i)) + |c^T B||s(t)| d_{\max} \leq -(\mu - \sigma)(||x(t_i)|| + \beta) |s(t)| + \frac{|\alpha|}{2} s^2(t) + \frac{|\alpha|}{2} s^2(t - d).$$
(11)

Now we claim that

$$|s(t)| \le (||c|| + ||A||^{-1})(||x(t_i)|| + \beta).$$
(12)

If (12) is not correct, then we have

$$(\| c\|+\| A\|^{-1})(\| x(t_i)\|+\beta) <| s(t) |\leq \| c\| \| x(t)\| \leq \| c\|(\| x(t_i)\|+\| e(t)\|) \\ \leq (\| c\|+\sigma\| A\|^{-1})\| x(t_i)\|+\sigma\beta\| A\|^{-1},$$

which is a contradiction. Thus we know (12) is correct. It then follows from (11) and the condition (8) that  $\dot{V}(t) \le -(\eta + \frac{|\alpha|}{2})s^2(t) + \frac{|\alpha|}{2}s^2(t-d)$ . According to Lemma 1, we obtain that as long as  $\operatorname{sign}(s(t_i)) = \operatorname{sign}(s(t))$ ,

$$V(t) \le \sup_{\theta \in [t_i - \tau, t_i]} V(\theta) e^{-\kappa(t - t_i)},$$
(13)

where  $\kappa > 0$  is the smallest real root of the equation  $\eta + \frac{|\alpha|}{2} - \kappa - \frac{|\alpha|}{2}e^{\kappa \tau} = 0$ , which means that the trajectories decrease before the sign of s(t) changes. The decrement of V(t) cannot be guaranteed since zero crossing of occurs. However, when the system is triggered at the time sequence  $\{t_i\}_{i=0}^{\infty}$ ,  $\operatorname{sign}(s(t_i)) = \operatorname{sign}(s(t))$  hold,  $i \in \mathbb{N}$ . Therefore there exists a practical sliding band such that V(t) decrease outside the region and  $\operatorname{sign}(s(t_i)) \neq \operatorname{sign}(s(t))$  in the region. Now we analyze the the size of the band, that is, the maximum deviation of sliding trajectory with zero crossing. The region is derived as follows

$$|s(t_i) - s(t)| = |c^T x(t_i) - c^T x(t)| \le ||c|| ||e|| \le (||x(t_i)|| + \beta)) ||A||^{-1}.$$

Then the maximum deviation of sliding trajectory can be obtained for the case  $|s(t_i)|=0$ , which yields (9). The proof is completed.

Now we study the boundedness of trajectories of the closed-loop system. Let  $x(t) = [x_1(t)^T, x_2(t)]^T$ ,  $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ ,  $B = [\mathbf{0}, B_2]^T$ , where  $x_1 \in \mathbb{R}^{n-1}$ ,  $x_2 \in \mathbb{R}$ ,  $A_{11} \in \mathbb{R}^{n-1 \times n-1}$ ,  $A_{12} \in \mathbb{R}^{n-1 \times 1}$ ,  $A_{21} \in \mathbb{R}^{1 \times n-1}$ ,  $A_{22} \in \mathbb{R}$ ,  $\mathbf{0} \in \mathbb{R}^{n-1}$ ,  $B_2 \in \mathbb{R}$ . Then we rewrite the system (1) in regular form:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + \alpha x_1(t-\tau),$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + \alpha x_2(t-\tau) + B_2u(t) + B_2d(t).$$
(14)
(15)

$$x_2 = A_{21}x_1 + A_{22}x_2 + \alpha x_2(t-\tau) + B_2u(t) + B_2u(t).$$
(1)

The following theorem shows that the ultimate boundedness of the trajectories.

**Theorem 2.** Consider the system (14) based on the event-triggered strategy (6) and control law (7).  $K(x(t_i))$  is still designed as (8) and  $A_{11} - A_{12}c_1^T + (\alpha + 1)I$  is Hurwitz. Then the trajectories of the system remain ultimately bounded in the region given as

$$\Omega = \left\{ x_1 \in \mathbb{R}^{n-1} \mid \| x_1 \| \le \sqrt{\frac{\| A_{12} P A_{12} \|}{\lambda_{\min}(P)(\lambda_{\min}(P^{-1}Q) + 1)}} \frac{\| x(t_i) \| + \beta}{\| A \|} \right\},$$
(16)

where P and Q are positive definite matrices satisfying

$$(A_{11} - A_{12}c_1^T)^T P + P(A_{11} - A_{12}c_1^T) + 2(\alpha + 1)P = -Q.$$

**Proof.** According to Theorem 1, we know that the system enters into the sliding band given by

(9). Then we have  $x_2 \le -c_1^T x_1 + (||x(t_i)|| + \beta) ||A||^{-1}$ . We construct the Lyapunov function  $V(t) = x_1^T(t)Px_1(t)$ . Compute the derivative of V (t) along the system trajectories of (14)

$$\begin{split} \dot{V}(t) &\leq x_{1}^{T} (A_{11}^{T}P + PA_{11} - PA_{12}c_{1}^{T} - c_{1}A_{12}^{T}P)x_{1}(t) + 2x_{1}^{T}PA_{12}(||x(t_{i})|| + \beta)||A||^{-1} \\ &+ \alpha x_{1}^{T}(t-\tau)Px_{1}(t-\tau) + \alpha x_{1}^{T}(t)Px_{1}(t) \\ &\leq -[\lambda_{\min}(P^{-1}Q) + \alpha + 1]V(t) + \alpha V(t-\tau) + ||A_{12}^{T}PA_{12}||(||x(t_{i})|| + \beta)^{2}||A||^{-2} \\ &\leq \Big[\sup_{\theta \in [t_{i}-\tau,t_{i}]} V(\theta) - \frac{||A_{12}PA_{12}||(||x(t_{i})|| + \beta)^{2}}{(\lambda_{\min}(P^{-1}Q) + 1)||A||^{2}}\Big]e^{-(\lambda_{\min}(P^{-1}Q) + 1)(t-t_{i})} + \frac{||A_{12}PA_{12}||(||x(t_{i})|| + \beta)^{2}}{(\lambda_{\min}(P^{-1}Q) + 1)||A||^{2}}. \end{split}$$

Therefore  $V(t) \le \frac{\|A_{12}PA_{12}\|(\|x(t_i)\| + \beta)^2}{(\lambda_{\min}(P^{-1}Q) + 1)\|A\|^2}$ , which yields that

$$||x_{1}(t)|| \leq \sqrt{\frac{1}{\lambda_{\min}(P)}V(t)} \leq \sqrt{\frac{||A_{12}PA_{12}||}{\lambda_{\min}(P)(\lambda_{\min}(P^{-1}Q)+1)}} \frac{||x(t_{i})|| + \beta}{||A||}.$$

The proof is complete.

As is well known, for a given initial condition, if the updating times of the controller converge to a finite constant, the event-triggered scheme induces undesired accumulation of event instants, Zeno phenomena. Now we prove that the Zeno behaviors are excluded in this paper.

**Theorem 3.** Consider system (1).  $\{t_i\}_{i=1}^{\infty}$  is the event-triggering time sequence generated by the triggering rule (6). If the conditions of Theorem 1 hold, then the time interval between any two consecutive event triggering instants has a lower bound  $T^*$  given as follows

$$T^{*} \geq \frac{1}{|\alpha| + ||A||} \ln \Big[ 1 + \frac{\sigma(||x(t_{i})|| + \beta)(|\alpha| + ||A||)}{||c|| ||A||(\phi(x(t_{i})) + \varsigma)} \Big],$$
(17)

where  $\phi(x(t_i)) = |c^T B|^{-1} || B|| (|K(x(t_i))| + || c|| || A|| || x(t_i)||), \quad \zeta = || B|| d_{\max}$ . **Proof.** Consider  $\Sigma = \{t \in [t_i - \tau, \infty) : || x(t) = 0||\}$ . For all  $t \in [t_i, t_{i+1}) \setminus \Sigma$ , we have

$$\frac{d\|e(t)\|}{dt} \le \|A\| \|x(t)\| + |\alpha| \|x(t-\tau)\| + |c^{T}B|^{-1} \|B\|(|K(x(t_{i}))| + \|c\| \|A\| \|x(t_{i})\|) + \|B\| d_{\max} \le \|A\| \|x(t)\| + |\alpha| \|x(t-\tau)\| + \phi(x(t_{i})) + \varsigma.$$

Then according to Lemma 2, we obtain  $t \in [t_i, t_i + 1)$ 

$$\| e(t) \| \le \| x(t) \| + \| x(t_i) \| \le \left[ \sup_{\theta \in [t_i - \tau, t_i]} \| x(\theta) \| + \frac{\phi(x(t_i)) + \varsigma}{|\alpha| + ||A||} \right] e^{(|\alpha| + ||A||)(t - t_i)} - \frac{\phi(x(t_i)) + \varsigma}{|\alpha| + ||A||} + \| x(t_i) \| \le \frac{\phi(x(t_i)) + \varsigma}{|\alpha| + ||A||} \left[ e^{(|\alpha| + ||A||)(t - t_i)} - 1 \right].$$

Note that there exists a minimum time interval for the error ||e(t)|| from 0 to  $\sigma(||x(t_i)|| + \beta)||c||^{-1}||A||^{-1}$ . Thus we have  $\frac{\sigma(||x(t_i)|| + \beta)}{||c|| ||A||} \le \frac{\phi(x(t_i)) + \zeta}{||\alpha| + ||A||} \left[e^{(|\alpha| + ||A||)T} - 1\right]$ , which yields the lower bounded of the event triggering time interval. The proof is complete

lower bounded of the event-triggering time interval. The proof is complete.

## 4. A Numerical Example

In this section, we give an example to illustrate the effectiveness of the obtain results. Consider the following delayed differential system.

$$\dot{x}(t) = \begin{bmatrix} 0 & 3\\ 4 & 7 \end{bmatrix} x(t) - 0.5x(t - 0.05) + \begin{bmatrix} 0\\ 1 \end{bmatrix} (u(t) + 0.2\sin(5t)).$$
(18)

Here we define the sliding variable as  $s(t) = c^T x(t) = [0.5 \ 1]x(t)$ . The gain function is defined as  $K(x(t_i)) = K_1 + K_2 ||x(t_i)||$ . The parameters are chosen as  $\beta = 0.5$ ,  $\sigma = 0.85$ ,  $K_1 = 1.5$ ,  $K_2 = 3$ . The

initial value is chosen as  $x_0 = \begin{bmatrix} 5 & 6 \end{bmatrix}^T$ . It can be derived that the gain condition (8) is satisfied. From the Theorem 1, the trajectory remains within a band and from Theorem 2 and the time interval between any two consecutive event triggering instants has a positive lower bound.



Fig. 1: State trajectory in phase plane.

Fig. 2: Dynamic behavior of x(t)w.r.t.time.

For simulation purpose, let the time interval be [0, 8s] and the step be 0.00125s. The simulation results for the delayed differential system with event-triggered sliding mode control are shown in Figure 1-4, which illustrate the performance of the global event-triggered sliding mode control. Figure 1 shows that the trajectory is attracted towards the sliding manifold S and stays within the practical sliding band which is dependent on the state. Figure 2 shows the dynamic behavior of  $x_1(t)$  and  $x_2(t)$  with respect to time. The event-triggered sliding mode control is shown in Figure 3. Figure 4 shows that the interval between any two consecutive event-triggered time is lower bounded

by a positive quantity, which is in accordance with Theorem 2.



#### 5. Conclusion

This paper has been investigated the global robust stability problem for linear delayed differential systems with uncertainties by using an event-triggered sliding mode control. A sliding mode control with event triggering scheme and the estimations of the practical sliding band which the trajectory remains at last have been provided. Then we have excluded Zeno phenomenon for proposed event-triggered scheme in this paper. At last, a numerical example has been given to illustrate the effectiveness of our results.

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